Modern Communication Systems (ENEE 3306)

Problem Set 3

M-ary Data Transmission

Problem 1

(a) Consider two arbitrary signals $s_1(t)$ and $s_2(t)$ whose energies are E_1 and E_2, respectively. Both signals are time-limited over $0 \leq t \leq T_b$. It is known that two *orthonormal* functions $\phi_1(t)$ and $\phi_2(t)$ can be used to represent $s_1(t)$ and $s_2(t)$ exactly as follows:

$$\begin{cases} s_1(t) = s_{11}\phi_1(t) + s_{12}\phi_2(t) \\ s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t) \end{cases} . \tag{P5.2}$$

Show that $E_i = s_{i1}^2 + s_{i2}^2$ by directly evaluating $\int_0^{T_b} s_i^2(t)\mathrm{d}t$, $i = 1, 2$.

(b) Let

$$\phi_1(t) = \begin{cases} \sqrt{2/T_b}\cos(2\pi f_c t), & 0 \leq t \leq T_b \\ 0, & \text{otherwise} \end{cases} , \tag{P5.3}$$

and

$$\phi_2(t) = \begin{cases} \sqrt{2/T_b}\sin(2\pi f_c t), & 0 \leq t \leq T_b \\ 0, & \text{otherwise} \end{cases} . \tag{P5.4}$$

Find the minimum value of frequency f_c that makes $\phi_1(t)$ and $\phi_2(t)$ orthogonal.

Remark The signal set considered in (b) is an important one in passband communication systems, not only binary but also M-ary.

Problem 2:

Consider the following two signals that are time-limited to $[0, T_b]$:

$$s_1(t) = V\cos(2\pi f_c t), \tag{P5.5}$$

$$s_2(t) = V\cos(2\pi f_c t + \theta), \tag{P5.6}$$

where $f_c = k/2T_b$ and k is an integer.

(a) Find the energies of both signals. Then determine the value of V for which both signals have an unit energy.

(b) Determine the correlation coefficient ρ of the two signals. Recall that

$$\rho = \frac{1}{\sqrt{E_1 E_2}} \int_0^{T_b} s_1(t)s_2(t)\mathrm{d}t. \tag{P5.7}$$

(c) Plot ρ as a function of θ over the range $0 \le \theta \le 2\pi$. What is the value of θ that makes the two signals orthogonal?

(d) Verify that the distance between the two signals is $d = \sqrt{2E}\sqrt{1-\rho}$. What is the value of θ that maximizes the distance between the two signals?

Problem 3:

(*This problem emphasizes the geometrical approach to signal representation*) Consider two signals $s_1(t)$ and $s_2(t)$ as plotted in Figure 5.41(b). The two orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$ in Figure 5.41(a) are chosen to represent the two signals $s_1(t)$ and $s_2(t)$, i.e.,

$$\begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}. \tag{P5.16}$$

(a) Determine the coefficients s_{ij}, $i, j \in \{1, 2\}$.

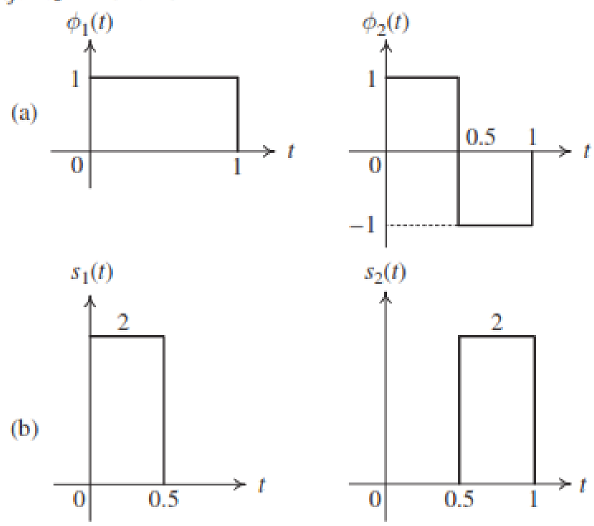


Fig. 5.41 (a) Orthonormal functions, and (b) signal set.

Problem 4:

(*Again more practice in determining an orthonormal basis set to represent the signal set exactly but in the M-ary, $M = 4$, case*) Consider the set of four time-limited waveforms shown in Figure 5.42.

(a) Using the Gram–Schmidt procedure, construct a set of orthonormal basis functions for these waveforms.

(b) By inspection, show that the set of orthonormal functions in Figure 5.43 can also be used to exactly represent the four signals in Figure 5.42.

(c) Plot the geometrical representation of the set of four signals $\{s_1(t), s_2(t), s_3(t), s_4(t)\}$ in the three-dimensional signal space spanned by $\{v_1(t), v_2(t), v_3(t)\}$.

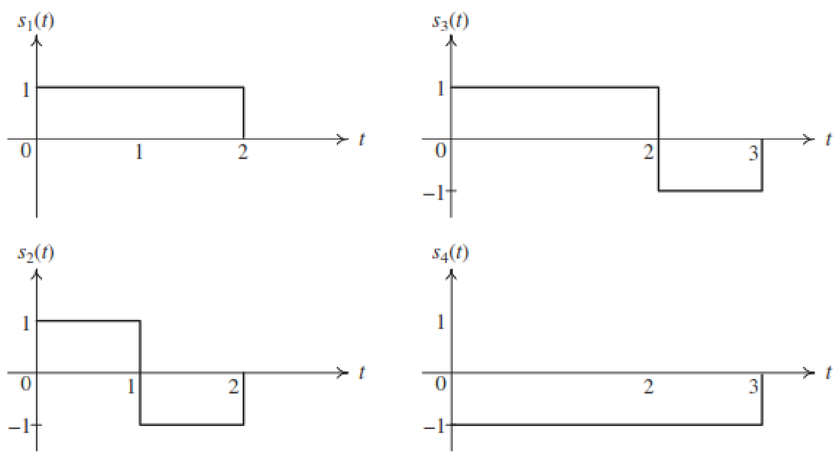
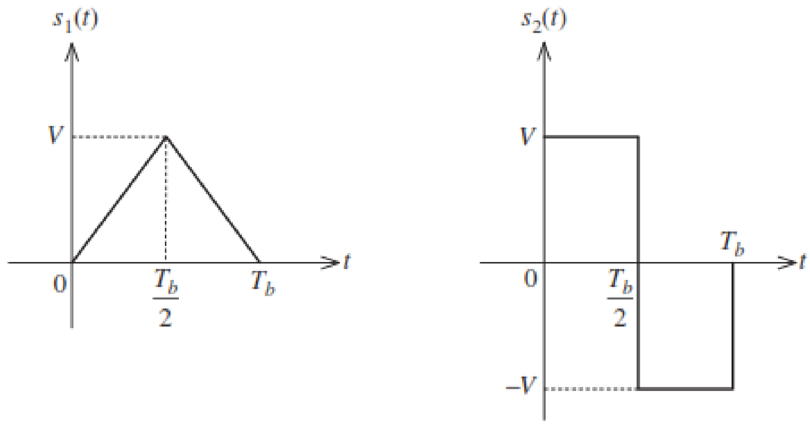


Fig. 5.42 A set of four time-limited waveforms.

Problem 5:

Consider the signal set in Figure 5.44 for binary data transmission over an AWGN channel. The noise is zero-mean and has two-sided PSD $N_0/2$. As usual, $s_1(t)$ and $s_2(t)$ are used for the transmission of equally likely bits "0" and "1," respectively.



(a) Show that $s_1(t)$ is *orthogonal* to $s_2(t)$. Then find and draw an orthonormal basis set $\{\phi_1(t), \phi_2(t)\}$ for the signal set.

(b) Draw the signal space diagram and the optimum decision regions. Write the expression for the optimum decision rule.

(c) Let $V = 1$ volt and assume that $N_0 = 10^{-8}$ watts/hertz. What is the maximum bit rate that can be sent with a probability of error $P[\text{error}] \leq 10^{-6}$.

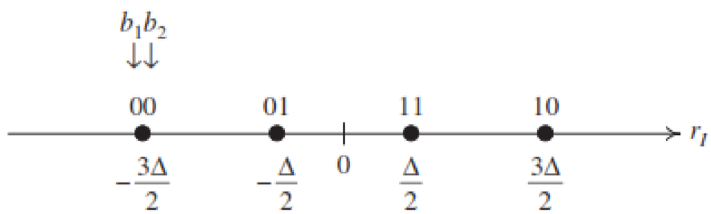
(d) Draw the block diagram of an optimum receiver that uses only one matched filter and sketch the impulse response of the matched filter.

(e) Assume that the signal $s_1(t)$ is fixed. However, you can change the shape, but not the energy, of $s_2(t)$. Modify $s_2(t)$ so that the probability of error is as small as possible. Explain your answer.

Problem 6:

To explore the influence of nearest neighbors consider the simple 4-ASK modulation with Gray mapping shown in Figure 8.27. The probability of symbol error is

$$P[\text{symbol error}] = \frac{2(M-1)}{M} Q\left(\frac{\Delta}{\sqrt{2N_0}}\right).$$



Signal constellation of 4-ASK with a Gray mapping.

(a) Determine the ratio $\Delta/\sqrt{2N_0}$ (which is related to the SNR $= E_b/N_0$) so that $P[\text{symbol error}] = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$.

(b) For each $P[\text{symbol error}]$ determine the following:

$$Q\left(\frac{\Delta}{\sqrt{2N_0}}\right),\ Q\left(\frac{3\Delta}{\sqrt{2N_0}}\right),\ Q\left(\frac{5\Delta}{\sqrt{2N_0}}\right).$$

(c) Consider that symbol 00 was transmitted. Determine the following error probabilities:

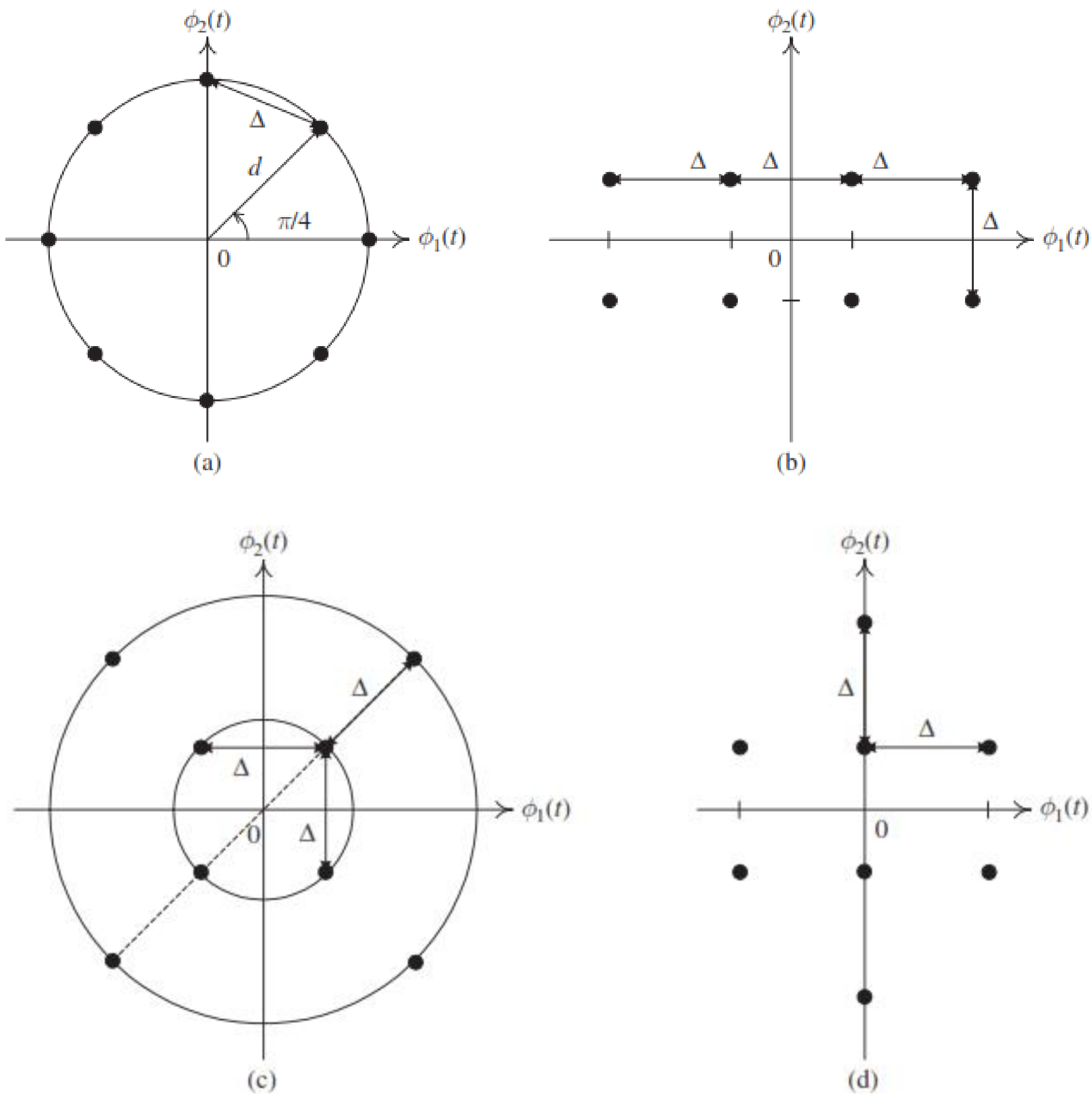
$$P\left[\{01\}_D \,|\, \{00\}_T\right],\ P\left[\{11\}_D \,|\, \{00\}_T\right],\ P\left[\{10\}_D \,|\, \{00\}_T\right].$$

(d) How would the answer in (c) change if one of the other symbols was chosen to be the transmitted one?
Therefore, in terms of neighbors the most important ones are the ________________ ones.

Problem 7:

(*8-ary constellations*) Consider the four 8-ary signal constellations in Figure 8.28, where all the signal points in each constellation are equally probable.

(a) Compute the average energies for the four constellations and rank the signal constellations in terms of energy efficiency.



The 8-ary constellations considered in Problem 8.9.

(b) Specify Gray mapping for the constellation in Figure 8.28(b).

(c) Draw the minimum-distance decision boundaries for the signal constellation in Figure 8.28(d). Which signals in this constellation are *most* susceptible to error and why?

Problem 8:

8.10 (*V.29 constellation*) The 16-QAM signal constellation shown in Figure 8.29 is an international standard for telephone-line modems, called V.29.

(a) Ignoring the four corner points at $(\pm 1.5\Delta, \pm 1.5\Delta)$, specify a Gray mapping of the constellation.

(b) Assume that all the 16 signal points are equally likely. Sketch the optimum decision boundaries of the minimum-distance receiver.

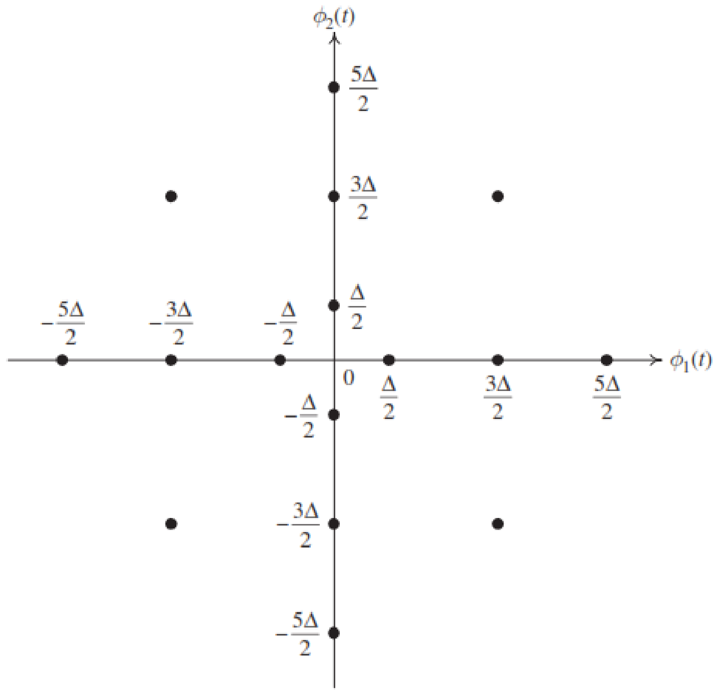


Fig. 8.29 V.29 constellation.

Problem 9:

8.11 (*16-QAM constellations*) Figure 8.30 shows two 16-QAM constellations.

(a) What can you say about the error performance of the two constellations? Which constellation is more energy-efficient? Explain.

(b) Specify a Gray mapping for constellation (b).

(c) Draw the minimum-distance decision boundaries for constellation (b). Which signals in this constellation are *least* susceptible to error and why?

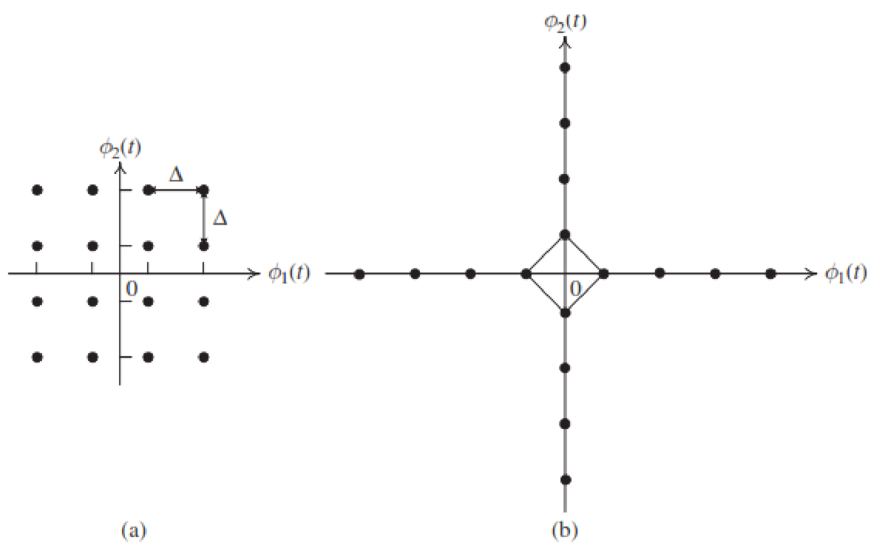


Fig. 8.30 The two 16-QAM constellations considered in Problem 8.11.